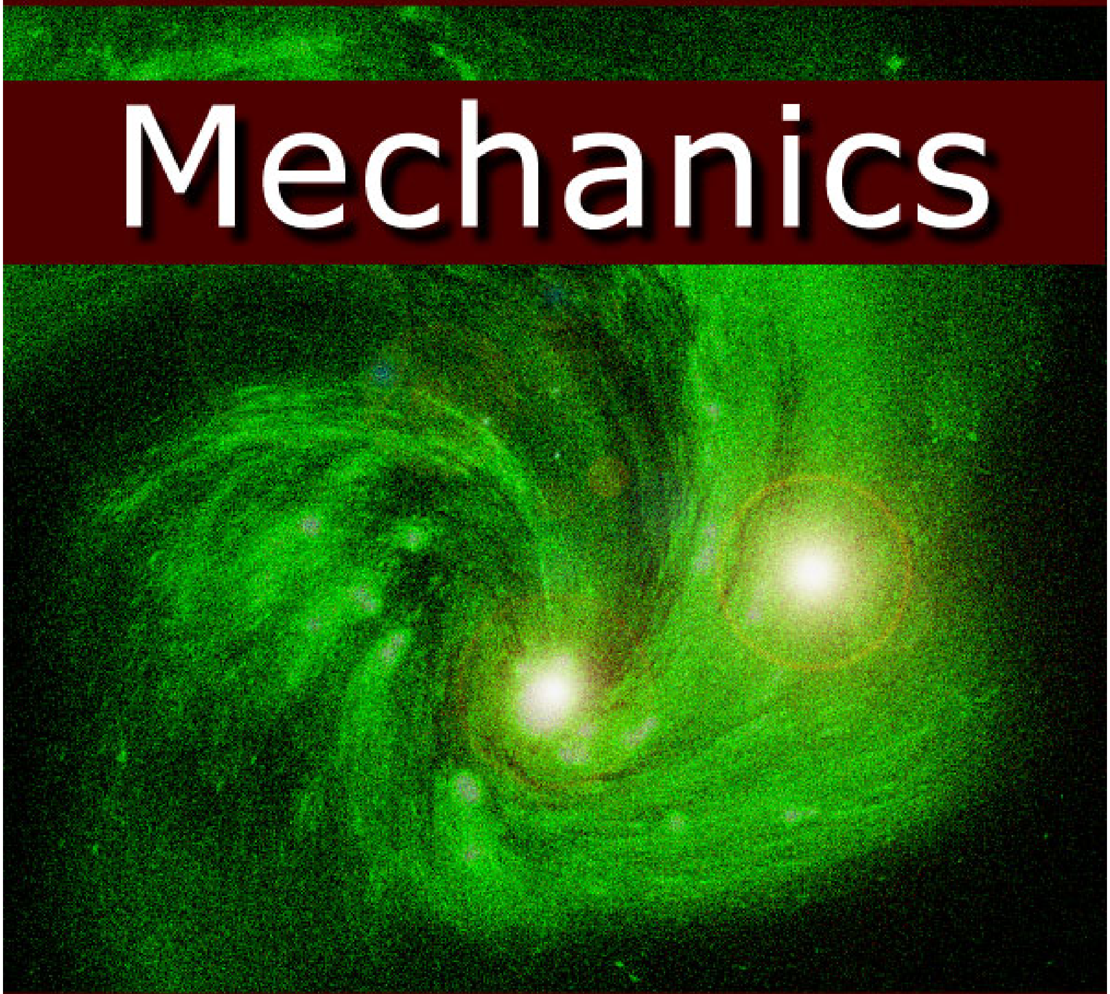
A-LEVEL MATHS TUTOR

Mechanics



PART FOUR

MOMENTUM IMPULSE

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Impulse

Momentum

Momentum is by definition the product of mass and velocity. So strictly speaking momentum is a vector quantity.

$$\text{momentum} = \text{mass(kg)} \times \text{velocity}(\text{ms}^{-1})$$

Hence the unit of momentum is (kg.ms^{-1}).

Impulse of a force

This is simply the force multiplied by the time the force acts.

We can obtain an expression for this in terms of momentum from Newton's Second Law equation F=ma, where the force F is constant.

Remembering that velocity, force and therefore impulse are vector quantities.

For a mass *m* being accelerated by a constant force **F**, where the impulse is **J** , $\mathbf{v}_1$ is initial velocity and $\mathbf{v}_2$ is final velocity:

$$\mathbf{F}t = m(\mathbf{v}_2 - \mathbf{v}_1)$$

$$\mathbf{J} = \mathbf{F}t$$

$$\mathbf{J} = m(\mathbf{v}_2 - \mathbf{v}_1)$$

Units

Since impulse is the product of force and time, the units of impulse are (Newtons) x (seconds), or N s .

Vector problems

Vector type questions on impulse are solved by first calculating the change in momentum. This gives a vector expression for the impulse. Using Pythagoras, the magnitude of the impulse can then be found. The anglular direction is calculated from the coefficients of unit vectors **i** and **j**.

Example #1

A particle of mass 0.5 kg moves with a constant velocity of (3**i** + 5**j**) $m.s^{-1}$.
After being given an impulse, the particle then moves off with a constant velocity of (2**i** - 3**j**) $m.s^{-1}$.

Calculate:

i) the impulse
ii) the magnitude of the impulse(to 2 d.p.)
iii) the direction of the impulse(θ degrees to the x-axis)

i)

$\mathbf{v}_1 = (3\mathbf{i} + 5\mathbf{j})$ $\mathbf{v}_2 = (2\mathbf{i} - 3\mathbf{j})$ $m = 0.5$ kg

using $\mathbf{J} = m(\mathbf{v}_2 - \mathbf{v}_1)$

$\mathbf{J} = 0.5(2\mathbf{i} - 3\mathbf{j}) - 0.5(3\mathbf{i} + 5\mathbf{j})$

$\mathbf{J} = \mathbf{i} - 1.5\mathbf{j} - 1.5\mathbf{i} - 2.5\mathbf{j}$

$\mathbf{J} = (1 - 1.5)\mathbf{i} + (-1.5 - 2.5)\mathbf{j}$

$\underline{\mathbf{J} = (0.5\mathbf{i} - 4\mathbf{j})\text{ N.s}}$

ii)

magnitude of impulse = $\sqrt{[(0.5)^2 + (-4)^2]} = \sqrt{[16.23]}$

= $\underline{4.03\text{ N.s}}$

iii) direction $\tan^{-1}\theta = (4)/(0.5) = 8$

$\theta = 82.8749^o = \underline{82.87}^o$ (2 d.p.) clockwise to the x-axis

Example #2

A particle of mass 2.5 kg is moving with a constant velocity of (2**i** + **j**) $m.s^{-1}$.
After an impulse, the particle moves with a constant velocity of (4**i** + 3**j**) $m.s^{-1}$.

Calculate:

i) the impulse
ii) the magnitude of the impulse(to 2 d.p.)
iii) the angle(θ°) the impulse makes with the x-axis

i)

$\mathbf{v}_1 = (2\mathbf{i} + \mathbf{j})$ $\mathbf{v}_2 = (4\mathbf{i} + 3\mathbf{j})$ $m = 2.5$ kg

using $\mathbf{J} = m(\mathbf{v}_2 - \mathbf{v}_1)$

$\mathbf{J} = 2.5(4\mathbf{i} + 3\mathbf{j}) - 2.5(2\mathbf{i} + \mathbf{j})$

$\mathbf{J} = 10\mathbf{i} + 7.5\mathbf{j} - 5\mathbf{i} - 2.5\mathbf{j}$

$\mathbf{J} = (10 - 5)\mathbf{i} + (7.5 - 2.5)\mathbf{j}$

$\underline{\mathbf{J} = (5\mathbf{i} + 5\mathbf{j})}$

ii)

magnitude of impulse = $\sqrt{[(5)^2 + (5)^2]} = \sqrt{[50]} = \underline{7.07 \text{ N.s}}$

iii) direction $\tan^{-1} \theta = (5)/(5) = 1$

$\theta = \underline{45^\circ}$ <u>anticlockwise to the x-axis</u>

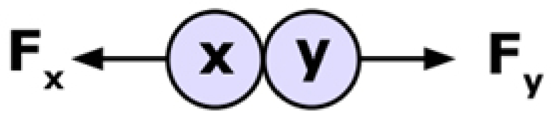
Conservation of Momentum

The Principle of Conservation of Momentum

The **total linear momentum** of a system of colliding bodies, with no external forces acting, **remains constant**.

for two perfectly elastic colliding bodies note:

i) By Newton's 3rd. Law, the force on X due to Y , (F_x) is the same as the force on Y due to X , (F_y) .



$F_x = F_y$

ii) By Newton's 2nd. Law, the rate of change of momentum is the same, since F = (rate of change of momentum)

iii)Because the directions of the momentum of the objects are opposite, (and therefore of different sign) the net change in momentum is zero.

Example #1

A 5 kg mass moves at a speed of 3 ms^{-1} when it collides head on, with a 3 kg mass travelling at 4 ms^{-1}, travelling along the same line.
After the collision, the two masses move off together with a common speed.

What is the common speed of the combined masses?

$m_1 = 5\text{ kg} \quad v_1 = 5\text{ ms}^{-1} \qquad m_2 = 3\text{ kg} \qquad v_2 = -4\text{ ms}^{-1}$

let the combined speed after collision be v_3

then, by the law of conservation of momentum,
momentum before collision = momentum after collision

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)v_3$$

$$\Rightarrow \qquad v_3 = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

substituting for m_1 v_1 m_2 and v_2

$$v_3 = \frac{(5 \times 3) + (3 \times (-4))}{5 + 3}$$

$$= \frac{15 - 12}{8} = \frac{3}{8} = 0.375$$

common speed of the two masses is 0.375 ms^{-1}

Example #2

An artillery shell of mass 10 kg is fired from a field gun of mass 2000 kg.
If the speed of the shell on leaving the muzzle of the gun is 250 ms^{-1} , what is the recoil speed of the gun?

$m_1 = 10\text{ kg} \quad v_1 = 250\text{ ms}^{-1} \quad m_2 = 2000\text{ kg}$

let the recoil speed of the gun after firing be v_2

then, by the law of conservation of momentum,

momentum before firing = momentum after firing

$$0 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow \quad -m_2 v_2 = m_1 v_1$$

$$\Rightarrow \quad v_2 = \frac{m_1 v_1}{-m_2}$$

substituting for $m_1 \quad v_1$ and m_2

$$v_2 = \frac{10 \times 250}{-2000} = -\frac{25}{20} = -1.25$$

(the minus signifies gun moves in opposite direction to the shell)

the speed of recoil of the gun is 1.25 ms^{-1}

Energy changes during collisions

Consider the kinetic energy change involved during a collision. Remember that no energy is actually lost, it is just converted into other forms. Energy can be transformed into heat, sound and permanent material distortion. The later causes the internal potential energy of bodies to increase.

If no kinetice energy is lost (K.E.= $\frac{1}{2}mv^2$) then the collision is said to be **perfectly elastic**. However if kinetic energy is lost, the collision is described as **inelastic**. In the special case when all the kinetic energy is lost, the collision is described as **completely inelastic**. This is when to two colliding bodies stick to one another on impact and have zero combined velocity.

The Coefficient of Restitution

The **Coefficient of Restitution (e)** is a variable number with no units, with limits from zero to one.

$$0 \le e \le 1$$

'e' is a consequence of **Newton's Experimental Law of Impact**, which describes how the speed of separation of two impacting bodies compares with their speed of approach.

note: the speeds are **relative** speeds

$$v_a = \text{speed of approach} \qquad v_s = \text{speed of separation}$$

$$e = \text{coefficient of restitution}$$

$$\text{coefficient of restitution} = \frac{\text{speed of separation}}{\text{speed of approach}}$$

$$e = \frac{v_s}{v_a}$$

If we consider the speed of individual masses before and after collision, we obtain another useful equation:

u_A = initial speed of mass A

u_B = initial speed of mass B

v_A = final speed of mass A

v_B = final speed of mass B

relative initial speed of mass A to mass B = $u_B - u_A$

relative final speed of mass A to mass B = $v_B - v_A$

$$\text{coefficient of restitution (e)} = \frac{v_B - v_A}{u_B - u_A}$$

note: in this equation the absolute of $u_B - u_A$ and $v_B - v_A$ are used (|absolute| no net negative result)

Example

A 5 kg mass moving at 6 ms^{-1} makes a head-on collision with a 4 kg mass travelling at 3 ms^{-1} .
Assuming that there are no external forces acting on the system, what are the velocities of the two masses after impact?
(assume coefficient of restitution e = 0.5)

u_A = initial speed of 5 kg mass (mass A)= 6 ms^{-1}

u_B = initial speed of 4 kg mass (mass B)= 3 ms^{-1}

m_A = 5 kg $\quad$ m_B = 4 kg

v_A = final speed of mass A $\quad$ v_B = final speed of mass B

momentum before the collision equals momentum after

hence, $\quad m_A u_A + m_B u_B = m_A v_A + m_B v_B$

also

$$\text{coefficient of restitution (e)} = \frac{v_B - v_A}{u_B - u_A}$$

substituting for e, m_A , u_A , m_B , u_B

we obtain two simultaneous equations

from the conservation of momentum,

$$5 \text{ x } 0.6 + 4(-3) = 5\, v_A + 4\, v_B$$

$$3 - 12 = 5\, v_A + 4\, v_B$$

$$-9 = 5\, v_A + 4\, v_B$$

$$5\, v_A + 4\, v_B = -9 \qquad \text{(i}$$

from the coefficient of restitution expression,

$$0.5 = \frac{v_B - v_A}{u_B - u_A}$$

$$0.5(u_A - u_B) = v_B - v_A$$

$$(0.5 \times 6) - (0.5(-3)) = v_B - v_A$$

$$3 + 1.5 = v_B - v_A$$

$$4.5 = v_B - v_A$$

$$v_B - v_A = 4.5 \qquad \text{(ii}$$

multiplying (ii by 5 and adding

$$5\ v_A + 4\ v_B = -9$$

$$-5\ v_A + 5\ v_B = 22.5$$

$$9\ v_B = 13.5$$

$$\boldsymbol{v}_B = 1.5\ ms^{-1}$$

from (ii

$$1.5 - v_A = 4.5$$

$$v_A = 1.5 - 4.5 = -3$$

$$\boldsymbol{v}_A = -3\ \text{ms}^{-1}$$

Ans. The velocities of the 5 kg and 4 kg masses are -3 $ms^{-1}$and 1.5 ms^{-1}, respectively.

Oblique Collisions

For two masses colliding along a line, Newton's Experimental law is true for component speeds. That is, the law is applied twice: to each pair of component speeds acting in a particular direction.

Example

A particle of mass *m* impacts a smooth wall at 4*u* ms^{-1} at an angle of 30 deg. to the vertical. The particle rebounds with a speed *ku* at 90 deg. to the original direction and in the same plane as the impact trajectory.

What is:
i) the value of the constant '*k*' ?
ii) the coefficient of restitution between the wall and the particle?
iii) the magnitude of the impulse of the wall on the particle

i) There is no momentum change parallel to the wall.

$$m(4u)\cos 30^\circ = m(ku)\cos 60^\circ$$

$$4u\frac{\sqrt{3}}{2} = ku.\frac{1}{2}$$

$$4\sqrt{3} = k, \qquad \underline{k = 4\sqrt{3}}$$

ii) The coefficient of restitution'e' is the ratio of the speed of separation to the speed of approach:

$$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{ku\sin 60^\circ}{4u\sin 30^\circ}$$

$$= \frac{4\sqrt{3}.\frac{\sqrt{3}}{2}u}{2u} = \frac{6u}{2u} = 3$$

$$\underline{e = 3}$$

iii) The impulse is the change of momentum.

Since the vertical unit vectors are unchanged, the momentum change just concerns the horizontal vector components.

hence,

$$\begin{aligned}\text{momentum change } &= m(4u\sin 30^\circ) - m(-ku\sin 60^\circ) \\ &= m.4u.\frac{1}{2} + m.4\sqrt{3}u\,\frac{\sqrt{3}}{2} \\ &= 2mu + 6mu \\ &= 8mu\end{aligned}$$

<u>ans. momentum change is $8mu$ N.s</u>

Notes