Worked Solutions

Pure Maths, Differential Calculus,

sheet PM_DIF_QR_01

The Quotient Rule Q.2

differentiate the function
$$y = \frac{4x}{x+3}$$

The Quotient Rule for differentiation states that for a function defined as:

$$y = \frac{u}{v}$$

where both u and v are differentiable functions of x, then the derivative of y with respect to x is given by:

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

let

$$u = 4x \qquad \text{and} \qquad v = x + 3$$

then,
$$\frac{du}{dx} = 4$$
 and $\frac{dv}{dx} = 1$

$$y = \frac{u}{v}$$

substituting into the Quotient Rule equation,

$$\frac{dy}{dx} = \frac{(x+3)\cdot 4 - 4x\cdot 1}{(x+3)^2}$$

simplifying,

$$\frac{dy}{dx} = \frac{4(x+3) - 4x}{(x+3)^2}$$

$$=\frac{4x+12-4x}{\left(x+3\right)^2}$$

$$=\frac{12}{(x+3)^2}$$

answer,

$$\frac{dy}{dx} = \frac{12}{\left(x+3\right)^2}$$