Worked Solutions

The Quotient Rule Q.6

differentiate the function $y = \dfrac{x^2 - 2}{x^2 + 3}$

The **Quotient Rule** for differentiation states that for a function defined as:

$$y = \frac{u}{v}$$

where both u and v are differentiable functions of x, then the derivative of y with respect to x is given by:

$$\frac{dy}{dx} = \frac{v \cdot \dfrac{du}{dx} - u \cdot \dfrac{dv}{dx}}{v^2}$$

let $u = x^2 - 2$ and $v = x^2 + 3$

then $\dfrac{du}{dx} = 2x$ and $\dfrac{dv}{dx} = 2x$

substituting into the Quotient Rule equation,

$$\frac{dy}{dx} = \frac{\left(x^2 + 3\right) \cdot 2x - \left(x^2 - 2\right) \cdot 2x}{\left(x^2 + 3\right)^2}$$

simplifying,

$$\frac{dy}{dx} = \frac{2x\left(x^2 + 3\right) - 2x\left(x^2 - 2\right)}{\left(x^2 + 3\right)^2}$$

$$= \frac{2x^3 + 6x - 2x^3 + 4x}{\left(x^2 + 3\right)^2}$$

$$= \frac{10x}{\left(x^2 + 3\right)^2}$$

answer,

$$\frac{dy}{dx} = \frac{10x}{\left(x^2 + 3\right)^2}$$