Worked Solutions

The Quotient Rule Q.7

differentiate the function $\qquad y = \dfrac{x^2+5}{4-x}$

The **Quotient Rule** for differentiation states that for a function defined as:

$$y = \frac{u}{v}$$

where both u and v are differentiable functions of x, then the derivative of y with respect to x is given by:

$$\frac{dy}{dx} = \frac{v \cdot \dfrac{du}{dx} - u \cdot \dfrac{dv}{dx}}{v^2}$$

let $\qquad u = x^2 + 5 \qquad$ and $\qquad v = 4 - x$

then $\qquad \dfrac{du}{dx} = 2x \qquad$ and $\qquad \dfrac{dv}{dx} = -1$

substituting into the Quotient Rule equation,

$$\frac{dy}{dx} = \frac{(4-x)\cdot 2x - \left(x^2+5\right)\cdot(-1)}{(4-x)^2}$$

simplifying,

$$\frac{dy}{dx} = \frac{2x(4-x) + \left(x^2+5\right)}{(4-x)^2}$$

$$= \frac{8x - 2x^2 + x^2 + 5}{(4-x)^2}$$

$$= \frac{-x^2 + 8x + 5}{(4-x)^2}$$

answer,

$$\frac{dy}{dx} = \frac{-x^2 + 8x + 5}{(4-x)^2}$$