

Worked Solutions

Pure Maths, Differential Calculus,

sheet PM_DIF_QR_01

The Quotient Rule Q.8

differentiate the function $y = \frac{x^3 + 1}{1 - x}$

The **Quotient Rule** for differentiation states that for a function defined as:

$$y = \frac{u}{v}$$

where both u and v are differentiable functions of x , then the derivative of y with respect to x is given by:

$$\frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

let $u = x^3 + 1$ and $v = 1 - x$

then $\frac{du}{dx} = 3x^2$ and $\frac{dv}{dx} = -1$

substituting into the Quotient Rule equation,

$$\frac{dy}{dx} = \frac{(1-x) \cdot 3x^2 - (x^3 + 1) \cdot (-1)}{(1-x)^2}$$

simplifying,

$$\frac{dy}{dx} = \frac{3x^2(1-x) + (x^3 + 1)}{(1-x)^2}$$

$$= \frac{3x^2 - 3x^3 + x^3 + 1}{(1-x)^2}$$

$$= \frac{-2x^3 + 3x^2 + 1}{(1-x)^2}$$

answer,

$$\frac{dy}{dx} = \frac{-2x^3 + 3x^2 + 1}{(1-x)^2}$$
