

Worked Solutions

Pure Maths, Differential Calculus,

sheet PM_DIF_QR_01

The Quotient Rule Q.9

differentiate the function $y = \frac{x^2 - 2}{5 - x^3}$

The **Quotient Rule** for differentiation states that for a function defined as:

$$y = \frac{u}{v}$$

where both u and v are differentiable functions of x , then the derivative of y with respect to x is given by:

$$\frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

$$\text{let } u = x^2 - 2 \quad \text{and} \quad v = 5 - x^3$$

$$\text{then } \frac{du}{dx} = 2x \quad \text{and} \quad \frac{dv}{dx} = -3x^2$$

substituting into the Quotient Rule equation,

$$\frac{dy}{dx} = \frac{(5 - x^3) \cdot 2x - (x^2 - 2) \cdot (-3x^2)}{(5 - x^3)^2}$$

simplifying,

$$\frac{dy}{dx} = \frac{2x(5 - x^3) + 3x^2(x^2 - 2)}{(5 - x^3)^2}$$

$$= \frac{10x - 2x^4 + 3x^4 - 6x^2}{(5 - x^3)^2}$$

$$= \frac{x^4 - 6x^2 + 10x}{(5 - x^3)^2}$$

answer,

$$\frac{dy}{dx} = \frac{x^4 - 6x^2 + 10x}{(5 - x^3)^2}$$
