

Worked Solutions

Pure Maths, Differential Calculus,

sheet PM_DIF_TF_01

Trigonometrical Functions Q.7

differentiate the function $y = \sin(x^2)$

The **Chain Rule** is used when differentiating a 'composite function', which is described as a function of another function.

The derivatives of the functions are linked by the equation:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

for $y = \sin(x^2)$, let u be the inner function, $u = x^2$
then the outer function is $y = \sin(u)$

taking derivatives with respect to u and y respectively,

$$\frac{du}{dx} = 2x \quad \text{and} \quad \frac{dy}{du} = \cos(u)$$

substituting these results into the Chain Rule equation,

$$\frac{dy}{dx} = \cos(u) \cdot 2x$$

substituting for $u = x^2$,

$$\frac{dy}{dx} = \cos(x^2) \cdot 2x$$

simplifying,

$$\frac{dy}{dx} = 2x \cos(x^2)$$
