Worked Solutions

Pure Maths, Differential Calculus, sheet PM-DIFF-CR-01

The Chain Rule Q. 10

The given expression is:

$$y = \left(2x^{-2} - x^3\right)^{1/3}.$$

We'll use the chain rule to find $\frac{dy}{dx}$:

Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx},$$

where $u = 2x^{-2} - x^3$.

Step 1: Define u and differentiate it with respect to x:

$$u = 2x^{-2} - x^3.$$

Differentiate u with respect to x:

$$\frac{du}{dx} = \frac{d}{dx}(2x^{-2}) - \frac{d}{dx}(x^3).$$

$$\frac{du}{dx} = -4x^{-3} - 3x^2.$$

Step 2: Differentiate y with respect to u:

$$y = u^{1/3}.$$

$$\frac{dy}{du} = \frac{1}{3}u^{-2/3}.$$

Step 3: Combine using the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

Substitute the expressions for $\frac{dy}{du}$ and $\frac{du}{dx}$:

$$\frac{dy}{dx} = \frac{1}{3}u^{-2/3} \cdot (-4x^{-3} - 3x^2).$$

Step 4: Substitute $u = 2x^{-2} - x^3$:

$$\frac{dy}{dx} = \frac{1}{3}(2x^{-2} - x^3)^{-2/3} \cdot (-4x^{-3} - 3x^2).$$