The Chain Rule Q. 4

differentiate $y = \left(2 - x^2\right)^2$.

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inner function is $u = 2 - x^2$

outer function is $y = u^2$,   where $u = 2 - x^2$

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differentiating the outer function $y = u^2$

$$\frac{dy}{du} = 2u$$

differentiating the inner function $u = 2 - x^2$

$$\frac{du}{dx} =- 2x$$

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using the Chain Rule: $$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

substituting for $$\frac{dy}{du} = 2u \quad \text{and} \quad \frac{du}{dx} = -2x$$

therefore $$\frac{dy}{dx} = 2u \cdot (-2x)$$

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substituting for $u = 2 - x^2$

$$\frac{dy}{dx} = 2\left(2 - x^2\right) \cdot (-2x)$$

$$\frac{dy}{dx} = -4x\left(2 - x^2\right)$$

answer: $$\frac{dy}{dx} = -8x + 4x^3$$

answer: $\dfrac{dy}{dx} = 4x^3 - 8x$