Worked Solutions

Pure Maths, Differential Calculus, sheet PM-DIFF-CR-01

The Chain Rule Q. 6

Step 1: Introduce u

Let $u = 1 + x - 3x^2$. Then the original equation becomes:

$$y = u^5$$

Step 2: Apply the Chain Rule

Using the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Step 3: Compute $\frac{dy}{du}$

From $y = u^5$, differentiate with respect to u:

$$\frac{dy}{du} = 5u^4$$

Step 4: Compute $\frac{du}{dx}$

From $u = 1 + x - 3x^2$, differentiate with respect to x:

$$\frac{du}{dx} = 1 - 6x$$

Step 5: Substitute Back

Now substitute $\frac{dy}{du}$ and $\frac{du}{dx}$ into the chain rule formula:

$$\frac{dy}{dx} = 5u^4 \cdot (1 - 6x)$$

Substitute $u = 1 + x - 3x^2$ back into the expression:

$$\frac{dy}{dx} = 5(1 + x - 3x^2)^4 \cdot (1 - 6x)$$

Final Answer:

$$\frac{dy}{dx} = 5(1 + x - 3x^2)^4(1 - 6x)$$