Worked Solutions

The Chain Rule   Q. 7

The given function is:

$$y = \left(3x^{-3} - 2x\right)^{1/2}.$$

We will use the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx},$$

where $u = 3x^{-3} - 2x$.

Step 1: Define u and differentiate it with respect to x:

$$u = 3x^{-3} - 2x.$$

Differentiate u with respect to x:

$$\frac{du}{dx} = \frac{d}{dx}(3x^{-3}) - \frac{d}{dx}(2x).$$

$$\frac{du}{dx} = -9x^{-4} - 2.$$

Step 2: Differentiate y with respect to u:

$$y = u^{1/2}.$$

$$\frac{dy}{du} = \frac{1}{2}u^{-1/2}.$$

Step 3: Combine using the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

Substitute the expressions for $\frac{dy}{du}$ and $\frac{du}{dx}$:

$$\frac{dy}{dx} = \frac{1}{2}u^{-1/2} \cdot (-9x^{-4} - 2).$$

Step 4: Substitute $u = 3x^{-3} - 2x$:

$$\frac{dy}{dx} = \frac{1}{2}(3x^{-3} - 2x)^{-1/2} \cdot (-9x^{-4} - 2).$$