Worked Solutions

Pure Maths, Differential Calculus, sheet PM-DIFF-CR-01

The Chain Rule Q. 8

We are tasked with differentiating $y = (3x - 2x^{-5})^{\frac{3}{5}}$ using the chain rule. Let's go step by step.

Step 1: Apply the chain rule

Let:

$$u = 3x - 2x^{-5}$$

Then:

$$y = u^{\frac{3}{5}}$$

The chain rule states:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Step 2: Differentiate $y = u^{\frac{3}{5}}$ with respect to u:

$$\frac{dy}{du} = \frac{3}{5}u^{-\frac{2}{5}}$$

Step 3: Differentiate $u = 3x - 2x^{-5}$ with respect to x:

$$\frac{du}{dx} = \frac{d}{dx}(3x) - \frac{d}{dx}(2x^{-5})$$

$$\frac{du}{dx} = 3 + 10x^{-6}$$

Step 4: Combine using the chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = \left(\frac{3}{5}u^{-\frac{2}{5}}\right) \cdot (3 + 10x^{-6})$$

Substitute back $u = 3x - 2x^{-5}$:

$$\frac{dy}{dx} = \frac{3}{5}(3x - 2x^{-5})^{-\frac{2}{5}} \cdot (3 + 10x^{-6})$$

Final Answer:

$$\frac{dy}{dx} = \frac{3(3 + 10x^{-6})}{5(3x - 2x^{-5})^{\frac{2}{5}}}$$