Worked Solutions

Pure Maths, Differential Calculus, sheet PM-DIFF-CR-01

The Chain Rule Q. 9

We are tasked to differentiate $f(x) = (4x^2 - x^{-3})^{\frac{1}{2}}$ using the chain rule. Let us proceed step by step.

Step 1: Use the chain rule

Let:

$$u = 4x^2 - x^{-3}$$

Then:

$$f(x) = u^{\frac{1}{2}}$$

The chain rule states:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Step 2: Differentiate $f(u) = u^{\frac{1}{2}}$ with respect to u:

$$\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$$

Step 3: Differentiate $u = 4x^2 - x^{-3}$ with respect to x:

$$\frac{du}{dx} = \frac{d}{dx}(4x^2) - \frac{d}{dx}(x^{-3})$$

$$\frac{du}{dx} = 8x + 3x^{-4}$$

Step 4: Combine using the chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = \left(\frac{1}{2}u^{-\frac{1}{2}}\right) \cdot (8x + 3x^{-4})$$

Substitute back $u = 4x^2 - x^{-3}$:

$$\frac{dy}{dx} = \frac{1}{2}(4x^2 - x^{-3})^{-\frac{1}{2}} \cdot (8x + 3x^{-4})$$

Final Answer:

$$\frac{dy}{dx} = \frac{8x + 3x^{-4}}{2\sqrt{4x^2 - x^{-3}}}$$